1. Sampling Inspection

The following describes the basic concept of the sampling inspection performed prior to the shipment of a product.

Assuming that the inspection method referred to as "sampling inspection by attributes" is used, and that the sample size is n and the acceptance number is C, the curve shown in Figure 1.1 is obtained, in which the horizontal axis represents the percentage of defects, while the vertical axis represents the probability of acceptance relative to the percentage of defects. This is called the operating characteristic (OC) curve and effectively represents the characteristics of a sampling inspection.

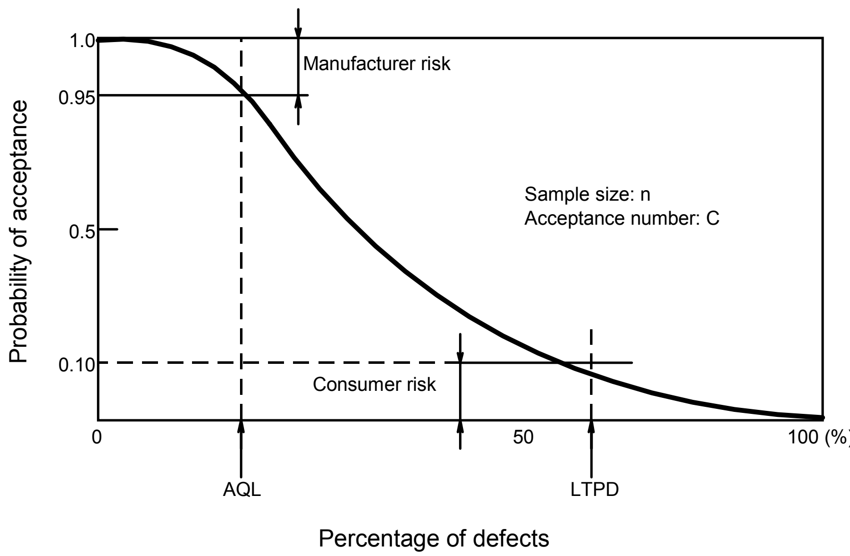

Figure 1.1 Operating characteristic (OC) curve

There are four important definitions associated with the OC curve for a sampling inspection:

(1) AQL (acceptable quality level):
The maximum percentage of defects for lots that may be accepted (specified by Toshiba).

(2) Manufacturer risk (α):
The probability of lots having a percentage of defects equal to the AQL.

(3) LTPD (lot tolerance percent defective):
The minimum percentage of defects for lots that may be rejected (specified by the customer).

(4) Consumer risk (β):
The probability of lots with the same defect percentage as the LTPD being accepted.

2. Mathematics of Reliability

2.1 Distributions in Reliability Analysis

The ultimate objective of any reliability study is to produce safe and reliable products. With this in mind, reliability studies can be divided into the following three major categories:

1. Studies of technological problems in improving reliability
2. Studies of control problems
3. Studies of problems involved in evaluating the results of 1 and 2.

The evaluation and quantification of reliability are prerequisites for selecting appropriate reliability improvements and control techniques, and are necessary for determining trade-offs with costs.

The evaluation procedure is as follows:

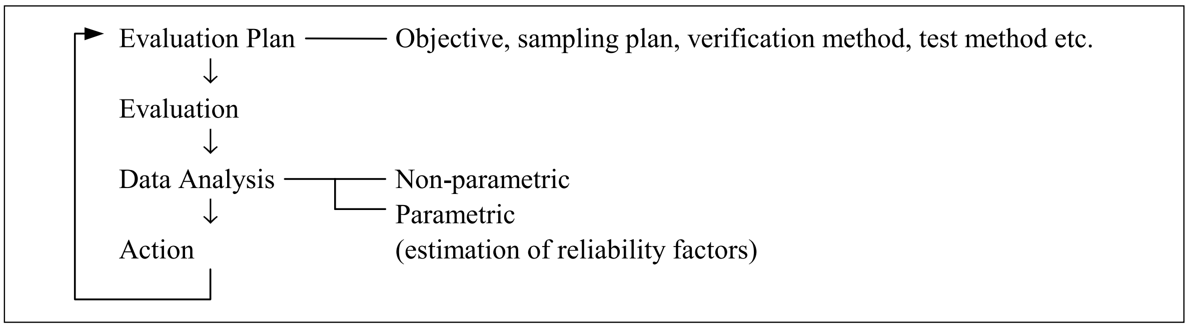

Figure 2.1 An example of evaluation procedure

This section describes the fundamental mathematics required for data analysis.

Data analysis is used to estimate reliability factors (such as the reliability level, mean lifetime and failure rate). The two estimation methods are the non-parametric method, which does not assume a distribution function, and the parametric method, which does assume a distribution function. The parametric method is prevalent because it is more precise and less costly, as described later. Both continuous distributions (the exponential, Weibull, log-normal, normal and gamma distributions) and discrete distributions (the geometric, binomial, Poisson and negative binomial distributions) are used.

2.1.1 Continuous Distributions

(1) Exponential distribution

The exponential distribution expresses the failure density function $f(t)$ as:

$f(t) = \lambda e^{-\lambda t}$ where λ is the failure rate (constant).

The reliability R(t) can be expressed as:

$R(t) = e^{-\lambda t}$

The failure rate is a constant (λ), independent of time, and the mean lifetime μ is:

$$\mu = \frac{1}{\lambda}$$

In other words, the reciprocal of the failure rate is the mean lifetime. The exponential distribution is characterized by the fact that the mean lifetime and MTBF are equal and by the fact that the reliability of the surviving product after a certain time has elapsed is equal to the initial reliability of the product.

(2) Weibull distribution

The failure density function $f(t)$ is given as:

$$f(t) = \frac{m(t-\gamma)^{m-1}}{t_O} \cdot e^{-\frac{(t-\gamma)^m}{t_O}}$$

and the failure rate λ (t), mean lifetime μ, reliability or survival rate R(t) at time t, and cumulative failure rate F(t) at time t are expressed as follows:

$$\lambda(t) = \frac{m\,(t-\gamma)^{m-1}}{t_0}$$

$$\mu = t_0^{\frac{1}{m}}\ \Gamma\left(1+\frac{1}{m}\right) \qquad (\Gamma = \text{gamma function})$$

$$R(t) = \exp\left\{-\frac{(t-\gamma)^m}{t_0}\right\}$$

$$F(t) = 1 - \exp\left\{-\frac{(t-\gamma)^m}{t_0}\right\}$$

In the above equation, m, t_0 and γ are distribution parameters. The parameter m determines the shape of the distribution and is referred to as the shape parameter. When the value of m is changed, the failure rate changes with time as shown in Figure 2.2. The distribution is exponential when m = 1. In other words, the Weibull distribution includes the exponential distribution as a special case. The failure rate increases with time when m > 1 and decreases when m < 1. When m is 3 or 4, the distribution is similar to the normal distribution which is described later. The parameter t_0 determines the time scale and is referred to as the parameter. γ determines the time at which failures start to occur and is referred to as the position parameter. When time $(t - \gamma) = t_O^{1/m}$ is substituted into the cumulative failure function, F(t) = 0.632, a constant value independent of m, $t_O^{1/m}$ and γ. Therefore, $t_O^{1/m}$ is referred to as the characteristic lifetime.

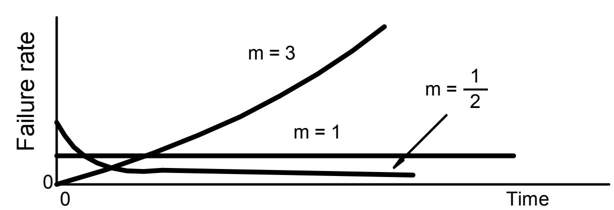

Figure 2.2 Relationship between the failure rate and the shape parameter m

(3) Log-normal distribution

In this distribution, the failure density function $f(t)$ is expressed as:

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma t} \exp\left\{-(1nt - m)^2 / 2\sigma^2\right\}$$

This becomes a normal distribution when 1nt = y.
The mean lifetime μ and median t_{50} are expressed respectively as:

$$\mu = \exp\left(m + \frac{1}{2}\sigma^2\right)$$

$$t_{50} = \exp \cdot \mu$$

where m is the average and σ the standard deviation of the distribution.

(4) Normal distribution

In this distribution, the failure density function $f(t)$, mean lifetime μ and failure rate λ(t) are expressed respectively as:

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} \cdot \exp\left\{-\frac{(t - m)^2}{2\sigma^2}\right\}$$

$$\mu = m$$

$$\lambda(t) = \frac{\exp\left\{-\frac{(t - m)^2}{2\sigma}\right\}}{\int_t^\infty \exp\left\{-\frac{(t - m)^2}{2\sigma^2}\right\} dt}$$

where m is the average and σ the standard deviation of the distribution.

(5) Gamma distribution

In this distribution, the failure density function $f(t)$, mean lifetime μ and failure rate λ(t) are expressed respectively as:

$$f(t) = \frac{m^k}{\Gamma(k)} t^{k-1} \cdot e^{-mt}$$

$$\mu = k/m$$

$$\lambda(t) = \frac{t^{k-t} e^{-mt}}{\int_t^\infty x^{k-1} \cdot e^{-mx} dx}$$

where k is called the shape parameter and λ is called the scale parameter. When k = 1, this distribution is similar to the exponential distribution; and when k is 4 or greater, it is similar to the normal distribution.

The gamma distribution can be considered a distribution of failures occurring for the first time after k harmful shocks have been received. In this case, m is the number of harmful shocks per unit time.

2.1.2 Discrete distributions

When it is physically impossible or inconvenient to inspect a product continuously until it fails, inspections are performed at specific intervals instead. In this case, time is not continuous and is treated as a discrete variable k (k = 0, 1, 2...). Distributions in which time is discrete are referred to as discrete distributions.

(1) Geometric Distribution

In this distribution, the failure density function $f(k)$ is expressed as:

$$f(k) = p \cdot q^{k-1} \quad (p + q = 1)$$

In this equation, p is the probability that a failure will occur (the failure rate) during the interval from time (k – 1) to time k, and is independent of the transition of time.

The mean lifetime μ and reliability R(k) are expressed respectively as:

$$\mu = 1/p$$
$$R(k) = q^k$$

When the time interval is made infinitely small, the result is an exponential distribution.

(2) Negative binomial distribution

The negative binomial distribution is a discrete form of the gamma distribution, just as the geometric distribution is a discrete form of the exponential distribution.

The failure density function $f(k)$, mean lifetime μ and reliability R(k) are expressed respectively as:

$$f(k) = \binom{k-1}{k-m} p^m q^{k-m} \qquad m=1, 2, \cdots \quad k\text{-}m, m+1, \cdots$$

$$\mu = m \cdot q/p$$

$$R(k) = 1 - \sum_{i=m}^{k} \binom{i}{k} p^i q^{k-1}$$

In the above equations, the parameters p and m can be considered as follows: p is the number of harmful shocks per unit time interval and m is the durability of the product against the shock. In other words, the product fails when harmful shocks are applied to the product m times.

(3) Compound negative binomial distribution

In (1) and (2) above, p was constant and was independent of time. If p is expressed as a function of time, as p(k), the failure density function $f(k)$ can be expressed as:

$$f(k) = \{1-p(1)\} \cdot \{1-p(2)\} \cdots\cdots\cdots\cdots \{1-p(k-1)\} \cdot p(k)$$
$$k = 1, 2 \cdots\cdots\cdots\cdots\cdots\cdots$$

which becomes a continuous Weibull distribution when

$$p(k) = \frac{\gamma}{\beta}\left\{k^{\beta} - (k-1)^{\beta}\right\}$$

(4) Binomial distribution

While the geometric distribution and negative binomial distribution are used to indicate reliability, the binomial distribution and the Poisson distribution (described below) are discrete distributions used mainly for sampling inspections.

The probability $P_B(r)$ of failure occurring r times during n tests is referred to as a binomial distribution and is expressed by the equation below. (Assuming that r failures occur when N items of the product are tested by inspecting n samples, if the relationship 10n < N exists, the probability of failure can be approximated by a binomial distribution.)

$$P_B(r) = \binom{n}{r} p^r (1-p)^{n-r}$$

In the above equation, p is the probability of failure occurring in a single test.

(5) Poisson distribution

When $np = \lambda$, $n \to \infty$ and $p \to 0$, the binomial distribution becomes a Poisson distribution with parameter λ.

PB(r)= (λ r/ri)e－ λ

where the parameter λ is equivalent to np in the binomial distribution. If n > 10 and p < 0.1, the Poisson distribution is applicable.

2.2 Estimation of Reliability

2.2.1 Non-Parametric Estimation of Reliability Scale

Various indices such as the reliability R(t) at time t, failure distribution function F(t), failure rate λ(t), and mean lifetime μ are used to quantify reliability, according to the situation.

Normally, each index is obtained after the lifetime distribution has been determined. However, sometimes it is necessary to determine the reliability without any knowledge of the lifetime distribution. The estimation method used in this case is referred to as the non-parametric method.

In non-parametric estimation, R(t) and F(t) can be expressed using the F distribution as follows:

$$\hat{R}(t) = 1/\{1+[(r+1)/(n-r)]F_{\alpha}, \upsilon 1, \upsilon 2\}$$

$$\hat{F}(t) = 1 - \hat{R}(t)$$

where

$\hat{R}(t)$ = estimated reliability after time t has elapsed

$\hat{F}(t)$ = estimated cumulative failure rate up until time t

r = number of failures that occurred during the test

n = number of products used for the test or the number of tests performed

F = upper α percentage point of F distribution corresponding to the variances υ_1 and υ_2

v1 = 2n − 2r

v2 = 2r + 2

1 − α = probability that the estimated reliability $\hat{R}(t)$ is equal to or greater than the true reliability. This is called the reliability level.

2.2.2 Estimating and Verifying the Lifetime Distribution Shape

(1) Estimation of distribution shape

The shape of the lifetime distribution is determined by first plotting the data obtained as a histogram, assuming a distribution from the shape of the histogram, and then verifying whether the assumption is correct. If the assumption is found to be incorrect, a different distribution is assumed and verification is attempted. These steps are repeated until the correct distribution is obtained. A probability paper can be used to estimate the distribution from a histogram. Normal, log-normal and Weibull probability papers are available. The paper that gives a straight line when the data is plotted with time on the horizontal axis and cumulative failure rate on the vertical axis indicates the applicable type of distribution (that is, the normal distribution can be applied if the plot is straight on normal distribution probability paper).

For example, let us plot a Weibull graph using six failures, with two failures at 1000h, one at 2000h, two at 3000h and one at 5000h for 1000 items of the tested product (see Figure 2.3). The data falls approximately on a straight line and the shape parameter m is 0.7. Therefore, this distribution can be considered a Weibull distribution.

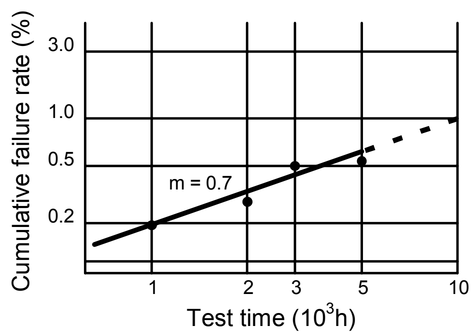

Figure 2.3 Maximum rating continuous operation test for TA7200P Series

(2) Verification of distribution shape

A method called χ^2 verification is used to confirm whether the distribution of a population is equal to the estimated distribution which is based on observed values.

Assume that the failure rate is f_i in each interval t_{i-1} to t_j when n items of the product are tested with the test time divided into k intervals (t_1, t_2, t_3, t_4....t_k). Next, the failure frequency p_i is obtained from the distribution to be verified. When

$$x = \sum_{i-m}^{k} (Fi - pi)^2 / pi$$

is substituted, if n is sufficiently large and $np_i > 10$, the distribution of x is approximated by a χ^2 distribution in which the variance $\phi = k - 1$.

In order to verify the assumption that the actual failure frequency occurring at each t_i is equal to the value obtained from the distribution to be verified, the value of
χ^2 α; φ which satisfies

$$P_r (\chi^2 \geq \chi^2 \alpha; \phi) = \alpha$$

is determined from the χ^2 table and compared with the χ^2 obtained.

If

$$x \leq \chi^2 \alpha; \phi$$

then the estimated distribution is correct. In the above equation, α is referred to as the level of significance of the statistics. In other words, the probability that the result of verification is incorrect is no more than α%. Usually a value of 5% or 10% is used.

If the distribution to be verified has m parameters, and if the parameters are estimated from data and the distribution is then verified, the variance φ of the χ^2 distribution is expressed as:

$$\phi = k - m - 1$$

2.2.3 Parametric Estimation of Reliability Scales

When the shape of the lifetime distribution is known, various indices required for reliability evaluation can be obtained by estimating the distribution parameters. The estimated parameters are themselves a function of the sampled values and form a certain distribution so that their values are different each time they are sampled, even if the population is identical. Therefore, the two evaluation methods are point estimation, in which the parameters are estimated at a single point, and interval estimation, in which they are estimated within a certain interval. An "Interval estimate of confidence level γ " means that the probability that the parameter for the population exists between interval estimates θ_L (lower estimate) and θ_U (upper estimate) is γ %. The confidence level is sometimes abbreviated to CL.

(1) Exponential distribution

(a) Fixed number testing method

In the fixed number testing method, testing is terminated when a predetermined number of failures occurs. The parameter for the exponential distribution λ (failure rate) is expressed as follows:

$$\bar{\lambda} = \frac{r}{\sum_{i-1}^{k} t_i + (n - r) t_r}$$

$$\lambda L = \frac{\chi^2_{1-\frac{\alpha}{2},2r}}{2r} \cdot \bar{\lambda}$$

$$\lambda U = \frac{\chi^2_{\frac{\alpha}{2},2r}}{2r} \cdot \bar{\lambda}$$

$\bar{\lambda}$ = point estimate of λ
λ_L = lower limit of λ interval estimate
λ_U = upper limit of λ interval estimate
n = number of tested samples
r = total number of failures
ti = time when the i-th failure occurred
$\chi^2\alpha, \phi$ = point where $P(\chi^2 \geq \chi^2\alpha, \phi) = \alpha$ in a χ^2 distribution with variance ϕ

The estimated values of the mean lifetime μ are expressed as follows:

$\bar{\mu} = 1/\bar{\lambda}$
$\mu_L = 1/\lambda_U$
$\mu_U = 1/\lambda_L$

where $\bar{\mu}$ =point estimate of mean

μ_L = lower limit of mean lifetime interval estimate
μ_U = upper limit of mean lifetime interval estimate

Furthermore, the point estimate of R(t) and interval estimate upper and lower limits are expressed as:

$\bar{R}(t) = e^{-\bar{\lambda}\cdot t}$

$\bar{R}_U(t) = e^{-\bar{\lambda}L\cdot t}$

$\bar{R}_L(t) = e^{-\bar{\lambda}U\cdot t}$

(b) Fixed time testing method
In the fixed time testing method, the test is terminated at a predetermined time t_C regardless of the number of failures. The point estimate and interval estimate of λ are expressed as follows:

$$\bar{\lambda} = \frac{r}{\sum_{i=1}^{k} t_i + (n-r) \cdot t_c}$$

$$\lambda L = \frac{\chi^2 1 - \frac{\alpha}{2}, 2r+2}{2r} \bar{\lambda}$$

$$\lambda U = \frac{\chi^2 \frac{\alpha}{2}, 2r+2}{2r} \cdot \bar{\lambda}$$

(2) Normal distribution

There are two normal distribution parameters: μ and σ^2. Parameter μ is the mean lifetime and σ^2 is the variance of the distribution. The point estimates of these values can be expressed as follows:

$$\bar{\mu} = \frac{\sum_{i=1}^{n} t_i}{n}$$

$$\bar{\sigma}^2 = \frac{\sum_{i=1}^{n} (t_i - \bar{\mu})^2}{n-1}$$

The upper and lower limits of the mean lifetime within the reliability interval μ_U to μ_L are:

$$\mu_U = \bar{\mu} + t_\alpha;(n-1) \cdot \sqrt{\frac{\bar{\alpha}^2}{n}}$$

$$\mu_L = \bar{\mu} - t_\alpha;(n-1) \cdot \sqrt{\frac{\bar{\sigma}^2}{n}}$$

and the upper and lower limits of the variance σ^2 within the reliability interval $\sigma^2{}_U$ to $\sigma^2{}_L$ are:

$$\sigma^2{}_L = \frac{(n-1)\,\bar{\sigma}^2}{\chi^2\ 1 - \frac{\alpha}{2}, n-1}$$

$$\sigma^2{}_U = \frac{(n-1)\,\bar{\sigma}^2}{\chi^2 \frac{\alpha}{2}, n-1}$$

where t_α, n is the value of t for which $P(t>t_\alpha, n) = \alpha$ in the t distribution table, and $\chi^2 \frac{\alpha}{2}, n-1$ is the value of χ^2 for which $P(\chi^2 \geq \chi^2 \frac{\alpha}{2}, n-1) = \frac{\alpha}{2}$ in the χ^2 distribution table.

(3) Weibull distribution

The Weibull distribution has three parameters, m, t_0 and γ, and it is very difficult to analyze data by calculation. Hence, the Weibull probability paper is often used for estimation. If m is known and $\gamma = 0$, t_0 is expressed as follows:

$$\bar{t_0} = \frac{\sum_{i=1}^{r} t_i^m + (n-r)\, t_r^m}{r}$$

(4) Log-normal distribution

There are two parameters μ and σ_2, as in the normal distribution. The point estimates are expressed as follows:

$$\bar{\mu} = (\sum_{i=1}^{n} \text{In} \quad ti)/n$$

$$\bar{\sigma}^2 = \sum_{i=1}^{n} (\text{In} \quad t_i - \bar{\mu})^2 / n - 1$$

and the estimate M of mean lifetime is expressed as follows:

$$\overline{M} = \exp(\bar{\mu} + \frac{\sigma^2}{2})$$

2.2.4 Using Probability Papers

Distribution analysis using a probability paper is very simple and requires no complicated calculations. The method is widely used in practice to verify theory in and for determining distribution parameters.

Various types of probability paper are available and their use is widely known. Described below are a few tips on how to plot data on probability paper and determine whether the result is a straight line. Many plotting methods have been devised to make the estimates as accurate as possible. The plot for a product for which n samples have been tested and for which the i-th product was defective is (t_i, F_i), where t_i is the time after which the i-th product failed and F_i is the cumulative failure rate.

The values

(1) i/n, (2) $(i - \frac{1}{2})/n$, (3) (i – 1)/(n – 1), (4) i/(n + 1) or (5) $(i - \alpha_i)/(n - \alpha_i - \beta_i + 1)$

are generally used to plot F_i.
(1) to (4) are very simple, but the last datum (that is, the nth datum) is not utilized in (1) and the first datum is not utilized in (3). Therefore (2) or (4) is recommended. Method (5) has been devised as an improvement over (4): $\alpha_i = \beta_i = \frac{3}{8}$ for the normal distribution; $\alpha_i = 0.52\ (1 - 1/m)$ and $\beta_i = 0.5 - 0.2\ (1 - 1/m)$ for the Weibull distribution with shape parameter m.

When determining the straightness of the curve, the conventional least squares method can be used. However, the data will not be distributed evenly and the variance will be smallest around the central part of the curve. Thus, when drawing the curve, make sure that it adheres closely to these central points.

2.3 Relationship between Failure Models and Life Distributions

2.3.1 Rope Model

The previous section described the mathematical methods for estimating the lifetime distribution. The lifetime distribution to be used can be further narrowed down if the relationship between the lifetime distribution and failure is known. When viewed from this perspective, the exponential distribution can be considered as a distribution of products which fail when subjected to m random harmful shocks per unit time. Similarly, the gamma distribution can be thought of as the case where a product receives k shocks before it fails.

Now assume that a product consists of many components, just as a rope consists of many strands. A rope fails when all of its strands are cut. Therefore, the following relationship exists between the reliability of a product and the reliability of its components:

$$R_D = 1 \prod_{i=1}^{k} (1 - R_i)$$

where R_D is the reliability of the product, R_i is the reliability of the i-th component and k is the number of components.

Provided that the lifetime distribution for each component forms an independent exponential distribution with the same shape, the distribution for the product as a whole is a gamma distribution, with a shape parameter k and a scale parameter m, which is the same shape as the exponential distributions for the components.

An item of equipment that will fail only when all of its components fail conforms to the “rope model” or “parallel model.” This model is used to study the problem of product fatigue and redundancy in design. When k in a gamma distribution becomes large, the distribution becomes similar to a normal distribution and the mean value becomes equal to k/m. Therefore, a normal distribution can be considered an extreme case of the rope model.

2.3.2 Weakest Link Model

In contrast to the rope model, a model similar to a chain of k links, where the failure of the weakest link results in the failure of the entire chain, is referred to as the “weakest link model.” This also applies to an item of equipment consisting of k components where the failure of any single component results in the failure of the equipment as a whole. For this reason, the weakest link model is also referred to as the “serial model.” In this case, the following relationship exists between the reliability R_D of the product and the reliability Ri of the components.

$$R_D = \prod_{i=1}^{k} R_i \qquad \text{k: number of compornents}$$

The Weibull distribution is one of the distributions that represent the weakest link model. In addition, the following double-exponential distribution, an extreme case of the Weibull distribution, is also used to represent the weakest link model.

$$F(t) = 1 - \exp(-\exp(t/n))$$

2.3.3 Proportional Effect Model

It is assumed that $X_1<X_2<X_3<\cdots<X_n$ are the fatigue cracks at each phase, and that the fatigue crack at each phase is proportional to that of the previous phase. In other words, if the following relationship exists

$$X_i = \alpha_i X_{i-1}$$

where α_i is a constant and i = 1, 2......n,
then the distribution of X_n is a log-normal distribution.

2.3.4 Stress and Strength Model

In this type of model, a product fails when stressed beyond its strength. In this model, failure can be calculated as the overlap of the stress distribution and the strength distribution.

If stress and strength are both normal distributions, the lifetime will also be a normal distribution.
If the average stress at a given time is μs and the standard deviation is σs, and similarly for the strength distribution, if μ_k is the average and σ_k is the standard deviation, then the cumulative failure is represented by the area of the normal strength distribution in which the strength is below 0, the average is equal to $(\mu_k - \mu_s)$ and the standard deviation is equal to $\sqrt{\sigma^2 k + \sigma^2 s}$

2.3.5 Reaction Rate Process Model

This model attempts to estimate the lifetime using a failure physics method. It assumes that a failure is caused at a microscopic level, where changes at the atomic and molecular levels cause harmful reactions and result in a failure when the changes reach a certain point. The following relations based on the Arrhenius model of chemical reaction are widely used.

$$\ln(L) = A + B/T - \alpha \ln(S)$$

L = mean lifetime
A, B, α = constants
T = temperature (K)
S = stress other than temperature

2.3.6 Reliability Model for Equipment

(1) Serial Model

For an article of equipment consisting of n components, if the equipment fails when one of its components fails, the reliability of the equipment $R_s(t)$ can be expressed as a function of the reliability of each component $R_i(t)$ as follows:

$$R_S(t) = 1 - \prod_{i=1}^{n} R_i(t)$$

(2) Parallel Model

For an article of equipment consisting of n' components running in parallel, with the equipment continuing to function as long as any of the parallel components is still running, the following equation is true:

$$R'_S(t) = 1 - \prod_{i=1}^{n'} (1 - R_i(t))$$

In this case, the reliability is better than that of equipment consisting of only one component.

[Bibliography]

Reliability Handbook, edited by W.G. Ireson (McGraw Hill)

Introduction to Reliability Engineering written by H.Shiomi (Maruzen Bookstore)